$c = 0$, has been reported by several authors, with some errors in derivation or transcription. For comparison purposes we record the correct equation here, with a key to the nomenclature used by various authors. The maximum (center line) velocity is U, and the channel half width is h. The following parameters are used:

$$W_1 = \lambda U/h \qquad W_2 = \Lambda U/h$$

$$B_1 = 1 + i\alpha(u_o - c)W_1 \qquad B_2 = 1 + i\alpha(u_o - c)W_2$$

$$A = \frac{B_1 - 1}{B_1} \qquad Re = hU\rho/\mu$$

u_o is the dimensionless parabolic velocity profile, $u_o = 1 - y^2$, and y is the dimensionless position. The stability equation for the eigenfunction ψ is then written (with $D = d/dy$, $u_o' = du_o/dy$, etc.):

$$i\alpha ReB_1[(u_o - c)(D^2 - \alpha^2) - u_o'']\psi = \sum_{n=0}^{4} b_n D^n\psi$$

where

$$b_4 = B_2$$

$$b_3 = 2i\alpha u_o'(W_1 - W_2)A$$

$$b_2 = -2\alpha^2 B_2 + (W_1 - W_2)A(3i\alpha u_o'' - 2\alpha^2 W_1 A u_o'^2)$$

$$b_1 = 2(W_1 - W_2)A\{i\alpha u_o''' - 2\alpha^2 W_1 A u_o' u_o'' - i\alpha^3(1 - 2W_1^2 u_o'^2/B_1)u_o'\}$$

$$b_o = \alpha^4 B_2 + (W_1 - W_2)\{i\alpha u_o'''' + \alpha^2(W_1/B_1)(3u_o''^2 + 4u_o' u_o''') - i\alpha^3 A(1 - 6W_1^2 u_o'^2/B_1)u_o'' + 2\alpha^4 W_1(1 + 1/B_1^2 + 2W_1^2 u_o'^2/B_1^2)u_o'^2\}$$

Here α is a Fourier wave number in the flow direction, and c is a complex wave velocity, such that $-\alpha c$ corresponds to λ_n in the section on stability and $\psi(y)\exp(i\alpha x)$ corresponds to $\phi_n(x)$. For $\lambda = \Lambda$, this reduces to the Newtonian stability equation. For the Maxwell fluid, we set $\Lambda = 0$ ($W_2 = 0$, $B_2 = 1$) and recover the published equation of Porteous and Denn (1972*a*). Their S is our B_1, the χ of Tlapa and Bernstein (1970), the τ of Tackels and Crochet (1973), and the β of Kundu (1972), whose γ is our B_2. The function q of Tackels and Crochet may be written $r/(r - 1)$ in their notation or $i\alpha W_1 u_o'/(B_1 - 1)$ in ours. Since the fluids we are considering have constant viscosities in steady shear, and hence have a parabolic profile, we could omit the terms in u_o''' and u_o''''.

The published equations of Tlapa and Bernstein contain four errors or misprints in their Equations (17) through (19), but they are not important for small W_1. In addition, Kundu reports two errors in their computer program and claims a larger disagreement than the neutral stability curves in the two papers would indicate. The other early attempt at deducing the stability equation for an Oldroyd fluid (Petrie, 1964) also suffers from a number of errors. The equations of Porteous and Denn and of Tackels and Crochet are consistent; the differences in the latter are apparently due to the neglect of some terms in the third and fourth derivatives of the unperturbed velocity, and these derivatives are zero for the fluid models considered.

THE AUTHORS

Chris Petrie went to Trinity College, Cambridge, and after graduating in Chemical Engineering in 1961 he was infected by the enthusiasm of J. R. A. Pearson and remained to write a Ph.D. dissertation on polymer flow instability. He has since applied his mathematical talents to engineering problems associated with polymer processing. At Newcastle, where he has lectured in engineering mathematics since 1966, his interest in mathematical modeling has also involved him in projects from sewage treatment to hemodialysis and from control engineering to chemistry. Away from work (and at other times) he enjoys fencing, bridge, swimming, and squash more than his ability would lead the onlooker to suspect, and from time to time he gets lost on mountains or falls off cliffs with the assistance of three energetic small boys and their long-suffering mother.

Morton Denn did his undergraduate work in chemical engineering at Princeton, where he gained his great respect for experimental rheologists while struggling to become one during senior thesis research with W. R. Schowalter. After graduation in 1961 he went to the University of Minnesota, where he received a Ph.D. in 1964 following thesis work with Rutherford Aris on a problem in optimization theory. He then returned to the fold, spending a post-doctoral year with A. B. Metzner's rheology and fluid mechanics research group at the University of Delaware. He joined the Delaware faculty in 1965, where his research has been in non-Newtonian fluid mechanics, stability, optimization, and control. His teaching focus has been on the development of undergraduate courses in chemical engineering analysis and fluid mechanics which emphasize the interaction between analysis, experiment, and engineering application, and on graduate courses related to his research interests. His noontime and weekend sport is swimming, and he is sometimes joined in this activity by his wife and three children.

Manuscript received November 10, 1975; revision received and accepted December 15, 1975.

A Theory of Isothermal Melt Spinning and Draw Resonance

The mechanics of isothermal melt spinning are studied for a viscoelastic liquid with a power law viscosity and a constant shear modulus. Steady state velocities and stresses are in agreement with experiment. The onset of the draw resonance instability, the magnitude of diameter fluctuations in the unstable region, and a second stable region at high draw ratio are predicted accurately.

ROBERT J. FISHER
and
MORTON M. DENN

Department of Chemical Engineering
University of Delaware
Newark, Delaware 19711

Correspondence concerning this paper should be addressed to Morton M. Denn. Robert J. Fisher is with Mobil Research and Development Corporation, Paulsboro, New Jersey 08540.

SCOPE

Melt spinning is the process of drawing an extruded liquid into a filament or sheet which is then solidified by rapid quenching or gradual cooling. The mechanics of this process have been studied extensively for low molecular weight inelastic liquids, but little has been done in analyzing spinning of high molecular weight molten polymers. Polymer melts show higher stress levels experimentally than inelastic liquids under comparable spinning conditions, and the initial rate of change of velocity with length is greater than in an inelastic liquid. Denn et al. (1975) have recently published an analysis of isothermal steady spinning of a viscoelastic liquid which has material properties that are independent of deformation rate. The results show the qualitative features observed in spinning experiments, but application to real spinning processes is severely limited, since the material properties of most melts of practical importance are highly deformation dependent. The first part of this paper is an extension of the previous results to fluids in which the viscosity is a power function of the deformation rate, but in which the shear modulus is a constant. This extension is an adequate material description for most polymers.

When the length of the molten filament is controlled by rapid quenching, an instability known as draw resonance sometimes occurs. This instability is characterized by a periodic variation in the diameter of the drawn filament. In most experiments the ratio of maximum to minimum diameter of the drawn filament deviates systematically from unity with increasing draw ratio (ratio of drawn filament velocity to initial velocity) beyond a critical value, but in one set of experiments the flow is stabilized at a draw ratio beyond the critical value by reduction of the spinning length. Draw resonance has been predicted by using theories for inelastic liquids, but the critical draw ratio computed in this way does not agree with experiments on molten polymers, and the inelastic theory does not predict the stabilization at high draw ratio. The latter is contained in an analysis by Zeichner (1973) for a viscoelastic liquid with a constant viscosity, but approximations in that analysis are not valid over much of the region of interest, and the results are only qualitatively correct. In this paper, draw resonance is analyzed by using linear and nonlinear stability theory for a viscoelastic liquid with a power law viscosity.

CONCLUSIONS AND SIGNIFICANCE

The steady state velocity profile and tension are in agreement with experiment on a shear thinning polystyrene melt, provided that the shear modulus in an extensional flow is taken to be 50 to 80% of its value as measured in a shear flow. This is consistent with results of a transient stretching experiment on the same polystyrene and with steady spinning experiments on low density polyethylene.

The stability calculations predict the onset of draw resonance at a critical draw ratio which is in agreement with experiment on several polymers. They also predict the stabilization at high draw ratio which is observed for short spinning lengths. Both the critical length at the upper stable region and the rate of deviation from unity of the ratio of maximum to minimum diameter are in quantitative agreement with experiments on polyethylene terephthalate.

The theory can thus be used for predictive purposes for both steady mechanics and stability calculations in spinning. The draw ratio at which draw resonance first occurs is insensitive to the shear modulus over a wide range, so the problem of estimating this parameter in an extensional flow is not a severe restriction. The point of stabilization at high draw ratio is quite sensitive to the modulus for highly shear thinning materials, however, so an accurate estimate would be needed there. The results point to a need to include a measure of the extensibility of the flow field in the stress constitutive equation.

In melt spinning a polymeric liquid is extruded through a spinneret or film die and is taken up downstream at a speed which is greater than the extrusion velocity. The process is shown schematically in Figure 1. The melt zone consists first of a region of die swell, caused by the relaxation of normal stresses generated during extrusion, followed by a region in which the molten filament is drawn down in area as the velocity increases to the downstream take-up velocity. The location of the point of solidification may be controlled precisely by rapid quenching with an air or water bath or chill roll, or solidification may be allowed to occur solely because of heat exchange with the surroundings all along the entire filament length.

The mechanics of steady spinning are not yet well understood. Rather complete theoretical analyses are available for low molecular weight inelastic liquids (Kase and Matsuo, 1965; Matovich and Pearson, 1969), but the stress levels observed in the spinning of polymer melts and solutions tend to be considerably higher, and the velocity variation with distance more linear, than these inelastic analyses would predict (for example, Kanel, 1972; Spearot and Metzner, 1972). Denn et al. (1975) studied the mechanics of isothermal steady spinning of a Maxwell fluid, which is a viscoelastic liquid with a constant viscosity and shear modulus. This model does predict high stresses and a linear velocity profile. It is inadequate for portraying the mechanics of most polymers quantitatively, however, because of the important role which the deformation-dependent shear viscosity plays in determining the kinematics and stresses.

The dynamics of melt spinning are also incompletely understood. When the length of the melt zone is fixed by a quench bath or chill roll, there is a critical draw ratio (ratio of take-up velocity to initial velocity) beyond which steady spinning cannot be maintained. This instability, known as draw resonance, is characterized by a periodic variation in the area of the drawn filament and an accompanying oscillation in tension. The first reported observations of draw resonance of which we are aware are by Christensen (1962) and by Miller (1963). There have been a number of studies since, and good quantitative data on a variety of polymers under isothermal and nearly isothermal conditions have recently

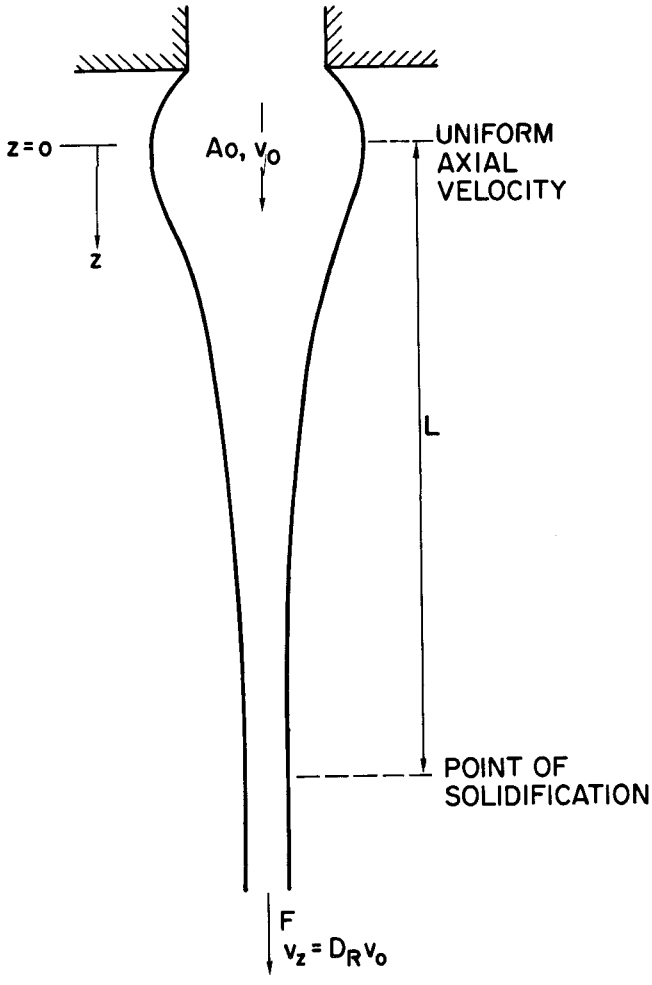

Fig. 1. Schematic of melt spinning.

been obtained by Zeichner (1973), by Ishihara and Kase (1975*b*), and by Weinberger and co-workers (Cruz-Saenz et al., 1975; Donnelly and Weinberger, 1975). Under certain spinning conditions there appears to be a damping of the oscillations and even complete stabilization at high draw ratio. This second stable region was reported by Lamb (1967) and is implicit in the data of Ishihara and Kase (1975*b*).

The onset of the instability has been calculated for inelastic liquids by Kase (1974; Kase et al., 1966) and by Pearson and co-workers (Pearson and Matovich, 1969; Pearson and Shah, 1974), but the results are not in agreement with many of the experiments cited above on the onset of the instability, and they do not predict stability at high draw ratio. An approximate analysis of the onset of draw resonance for a Maxwell fluid by Zeichner (1973; Denn, 1975) is in rough agreement with experiment for constant viscosity liquids, including a prediction of the second stable region, but the analysis contains approximations which are invalid under some processing conditions and is inapplicable in any event to liquids with a deformation-dependent viscosity. The nonlinear dynamics of draw resonance in the unstable region have been studied for inelastic liquids by Ishihara and Kase (1975*a*, *b*) and by Fisher and Denn (1975). These analyses show the qualitative features of draw resonance and demonstrate that the phenomenon must occur with any liquid that can sustain a tensile stress. The results are in agreement with experimental data only in the range where fluid elasticity is unimportant, however.

This paper reports on a solution of both the steady state and dynamical responses for isothermal melt spinning of a viscoelastic liquid with a deformation rate-dependent viscosity. The results are in good agreement with steady state and dynamical experimental data, and they provide a basis for prediction of stress levels, regions of stable processing, and the magnitude of oscillations in the unstable regime.

RHEOLOGICAL CHARACTERIZATION

The essential difficulty in analyzing the motion of a viscoelastic liquid is the choice of the constitutive equation relating stress to deformation. There is always a compromise between generality of the stress equation and ease of solution, and the type of process involved imposes restrictions on the class of constitutive equations which can be considered (for example, Astarita and Denn, 1975). Spinning can be a high Deborah number process, in which the residence time is comparable to the fluid relaxation time. For such a process the stress equation must account properly for the fluid memory over the entire process time.

The deformation dependence of the viscosity function is an important factor in polymer melt flow, and the stress equation used in this work is the simplest isothermal constitutive equation which both accounts for the shear thinning of the viscosity and is applicable to high Deborah number processes. We use an equation sometimes known as the White-Metzner model, which is a generalization of the Maxwell fluid, relating the extra stress τ to the deformation gradient as follows:

$$\tau + \frac{\mu(II_d)}{G}\left[\frac{\partial \tau}{\partial t} + \mathbf{v}\cdot\nabla\tau - (\nabla\mathbf{v})\cdot\tau - \tau\cdot(\nabla\mathbf{v})^T\right] = \mu(II_d)\,[\nabla\mathbf{v} + (\nabla\mathbf{v})^T] \quad (1)$$

The viscosity is a function of the second invariant of the deformation rate tensor

$$II_d = \text{trace}\,\{[\nabla\mathbf{v} + (\nabla\mathbf{v})^T]\cdot[\nabla\mathbf{v} + (\nabla\mathbf{v})^T]\} \quad (2)$$

while the shear modulus G is taken as a constant. The apparent relaxation time for the material is $\mu(II_d)/G$. The shear modulus is not truly constant for most polymeric liquids, but its variation with deformation rate is usually much less than the variation of the viscosity. A variable shear modulus could easily be incorporated, but the additional complexity would not introduce any important new physical phenomena and seems unwarranted.

The viscosity function for most polymer melts can be represented over several decades of deformation rate by the power law, or Ostwald-deWaele model

$$\mu(II_d) = K\left(\frac{1}{2}II_d\right)^{\frac{n-1}{2}} \quad (3)$$

and we shall use this representation for the analysis which follows. In a simple shear flow this viscosity function simplifies to the more familiar power law form

$$\mu(\dot{\gamma}_s) = K|\dot{\gamma}_s|^{n-1} \quad (4)$$

where $\dot{\gamma}_s$ is the shear rate. The power law index n ranges in value from about one-third for a highly shear thinning melt like polystyrene to a value of nearly unity for polyethylene terephthalate (PET), which has a relatively constant viscosity.

SPINNING EQUATIONS

It is not possible at present to analyze the region of die swell, and the origin of the coordinate system is taken approximately at the point of maximum diameter, where the axial velocity has become essentially uniform over the cross section. The equations for conservation of mass and energy can then be written (Kase and Matsuo, 1965; Matovich and Pearson, 1969):

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial z}(Av_z) = 0 \tag{5}$$

$$\frac{\partial}{\partial z}[A(\tau_{zz} - \tau_{xx})] = 0 \tag{6}$$

Here, z is the axial (spinning) direction, and x is any direction in the cross-sectional plane. These equations neglect inertia, gravity, and surface tension, all of which are usually negligible in melt spinning. The filament curvature is assumed to be small, in which case the isotropic pressure is numerically equal to the extra stress τ_{xx}. Thus, Equation (6) is simply a statement that the tension is the same at all points along the filament.

The x and z components of the constitutive equation are

$$\tau_{xx} + \frac{3^{\frac{n-1}{2}}K}{G}\left(\frac{\partial v_z}{\partial z}\right)^{n-1}\left[\frac{\partial \tau_{xx}}{\partial t} + v_z\frac{\partial \tau_{xx}}{\partial z} + \frac{\partial v_z}{\partial z}\tau_{xx}\right] = -3^{\frac{n-1}{2}}K\left(\frac{\partial v_z}{\partial z}\right)^n \tag{7}$$

$$\tau_{zz} + \frac{3^{\frac{n-1}{2}}K}{G}\left(\frac{\partial v_z}{\partial z}\right)^{n-1}\left[\frac{\partial \tau_{zz}}{\partial t} + v_z\frac{\partial \tau_{zz}}{\partial z} - 2\frac{\partial v_z}{\partial z}\tau_{zz}\right] = 2(3)^{\frac{n-1}{2}}K\left(\frac{\partial v_z}{\partial z}\right)^n \tag{8}$$

In simplifying Equation (1) it is assumed that the velocity v_z is uniform over the cross section and that terms of order A/L^2 are small compared to unity. It then follows that τ_{zz} and τ_{xx} are functions only of z and t.

This set of equations requires four spatial boundary conditions. The conditions at the origin, $z = 0$, are not evident because of the unknown behavior in the die swell region. The extrusion rate Q is assumed always to be constant. In a steady experiment the die swell area can be measured, or it can be estimated by using the very approximate approach of White and Roman (1975), so that at steady state the initial area and velocity are known at least in principle. Thus we have

$$\text{at } z = 0,\ v_z = v_o \tag{9a}$$

$$A = A_o \tag{9b}$$

The die swell does not appear to be greatly affected by transient conditions downstream, so we assume that these two boundary conditions hold at all times for both steady and transient processing. This is probably the most serious assumption in the analysis.

The velocity of the take-up device can be fixed. If we assume that the filament is not drawn in the solid region following the quench, or that it is quenched directly on the take-up roll, then the velocity at the end of the melt zone $z = L$ is known. The draw ratio D_R is defined as the ratio of take-up velocity to initial velocity, so the boundary condition can be written

$$\text{at } z = L,\ v_z = D_R v_o \tag{9c}$$

Under steady spinning conditions the draw ratio is also equal to the ratio of initial area to take-up area.

Finally, we need to establish some boundary condition for the stress. The initial values of τ_{zz} and τ_{xx} depend on both the prior processing in the spinneret and on the imposed tension on the spin line. Specification of both τ_{zz} and τ_{xx} at $z = 0$, or even the difference between the two, is not possible, since the product $A(\tau_{zz} - \tau_{xx})$ is the tension, and A_o is already fixed by assumption. The take-up velocity and the tension cannot both be specified; if the take-up velocity is fixed, then we must accept whatever tension is necessary to draw the filament. Thus, we take the fourth boundary condition as

$$\text{at } z = 0,\ \tau_{zz} = \tau_o \tag{9d}$$

τ_o is known only in the limit $\mu/G \to 0$, but its value turns out to be relatively unimportant in defining the significant features of melt spinning.

It is convenient to work in terms of dimensionless variables, defined as follows:

$$\xi = z/L \qquad \theta = t\,v_o/L$$
$$u = v_z/v_o \qquad a = A/A_o$$
$$T = \tau_{zz}Q/F\,v_o \qquad P = \tau_{xx}Q/F\,v_o$$

F is the force required under steady state conditions to draw the filament. Equations (5) through (9) then become

$$\frac{\partial a}{\partial \theta} + \frac{\partial}{\partial \xi}(au) = 0 \tag{10}$$

$$\frac{\partial}{\partial \xi}[a(T - P)] = 0 \tag{11}$$

$$P + \alpha\left(\frac{\partial u}{\partial \xi}\right)^{n-1}\left[\frac{\partial P}{\partial \theta} + u\frac{\partial P}{\partial \xi} + P\frac{\partial u}{\partial \xi}\right] = -\epsilon\left(\frac{\partial u}{\partial \xi}\right)^n \tag{12}$$

$$T + \alpha\left(\frac{\partial u}{\partial \xi}\right)^{n-1}\left[\frac{\partial T}{\partial \theta} + u\frac{\partial T}{\partial \xi} - 2T\frac{\partial u}{\partial \xi}\right] = 2\epsilon\left(\frac{\partial u}{\partial \xi}\right)^n \tag{13}$$

$$\text{at } \xi = 0,\ u = a = 1,\ T = T_o$$
$$\text{at } \xi = 1,\ u = D_R \tag{14}$$

T_o is the ratio of the initial value of the extra stress τ_{zz} to the total axial stress $\tau_{zz} - \tau_{xx}$. For an inelastic liquid, $T_o = 2/3$. For an elastic liquid with $n = 1$, the ratio $|\tau_{xx}/\tau_{zz}|$ at high stretch rates is small over most of the spin line regardless of the value of T_o (Denn et al., 1975), and the behavior for $T_o \neq 1$ is not very different from $T_o = 1$. $T_o = 1$ corresponds to the stress distribution for fully developed laminar flow in the spinneret.

The dimensionless groups α and ϵ are defined as

$$\alpha = \frac{3^{\frac{n-1}{2}}K}{G}\left(\frac{v_o}{L}\right)^n \qquad \epsilon = \frac{3^{\frac{n-1}{2}}KQ}{FL}\left(\frac{v_o}{L}\right)^{n-1}$$

α is the primary viscoelastic parameter, while ϵ is a reciprocal dimensionless force. For $n = 1$, these reduce to the groups defined by Denn et al. (1975). The ratio ϵ/α is of particular physical significance; it can be written as

$$\frac{\epsilon}{\alpha} = \frac{G}{F/A_o} \tag{15}$$

Thus, it represents the ratio of the characteristic stress of the material to the initial imposed stress.

STEADY STATE

At steady state the time derivatives $\partial/\partial\theta$ are zero, and Equations (10) through (13) reduce to a set of ordinary differential equations. These can be manipulated into a single, nonlinear, second-order differential equation for u with an accompanying algebraic equation defining T:

$$T_s = \frac{u_s}{3\alpha\left(\frac{du_s}{d\xi}\right)^n} + \frac{2}{3}u_s - \frac{\epsilon}{\alpha} \tag{16}$$

$$u_s + (\alpha u_s - 3\epsilon)\left(\frac{du_s}{d\xi}\right)^n - 2\alpha^2 u_s\left(\frac{du_s}{d\xi}\right)^{2n} - n\alpha u_s^2\left(\frac{d^2u_s}{d\xi^2}\right)\left(\frac{du_s}{d\xi}\right)^{n-2} = 0 \tag{17}$$

$$u_s(0) = 1,\ u_s(1) = D_R,\ T_s(0) = T_o \tag{18}$$

The subscript s here denotes steady state.

These equations reduce identically for the special case $n = 1$ to those studied by Denn et al. (1975). (Denn et al. considered a slight generalization involving an additional parameter ν, the ratio of normal stress differences. Setting $\nu = 0$ causes no fundamental change in behavior). The behavior is qualitatively the same for $n \neq 1$ as for $n = 1$. As $\alpha \to 0$, the equations for spinning an inelastic power law fluid are recovered (Pearson and Shah, 1974):

$$\alpha = 0:\ u_s(\xi) = \{1 + [D_R^{\frac{n-1}{n}} - 1]\xi\}^{\frac{n}{n-1}} \tag{19}$$

$$\epsilon = \frac{1}{3}\left\{\frac{n-1}{n[D_R^{\frac{n-1}{n}} - 1]}\right\}^n \tag{20}$$

At the other extreme, $\epsilon/\alpha \to 0$, the equations predict high stresses and a nearly linear velocity profile. The influence of T_o is important only close to $\xi = 0$, and T_o can be set equal to unity without seriously affecting the results. In that case the limiting profile is

$$\frac{\epsilon}{\alpha} = 0:\ u_s(\xi) = 1 + \frac{\xi}{\alpha^{1/n}} \tag{21}$$

For a given α there is a maximum attainable draw ratio beyond which the stresses are required to become infinite and processing is not possible. This limit is given approximately by

$$D_R \leq 1 + \alpha^{-1/n} \tag{22}$$

and corresponds to drawing the filament down in one relaxation time. The behavior is shown in Figures 2 and 3 for $n = 1/3$, $D_R = 5.85$, and by taking $T_o = 1$ for all values of α except $\alpha = 0$. (For $\alpha = 0$, the fluid has no memory, and T_o necessarily has the value 2/3). An interesting feature of this solution is that the apparent extensional, or Trouton viscosity, is a decreasing function of stretch rate (extension thinning) for small α, but it will increase with stretch rate (extension thickening) for sufficiently large α. Both types of behavior are observed in polymer melts (for example, Cruz-Saenz et al., 1975), and it is evident from this result that even the functional form of the apparent extensional viscosity depends on both the polymer and the processing conditions.

The applicability of the solution of the spinning equations for $n = 1$ has been tested in the high stress limit by Denn et al. (1975) on low density polyethylene data of Spearot and Metzner (1972). They found that the modulus G for the extensional flow field must be taken

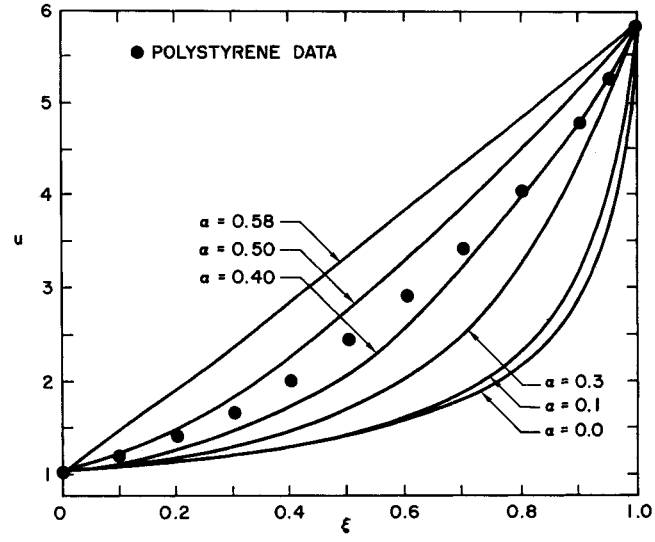

Fig. 2. Velocity profile for various values of α, $n = 1/3$, $D_R = 5.85$, $T_0 = 1$. Data of Zeichner (1973) on a polystyrene melt, Dow Styron 666, at 170°C.

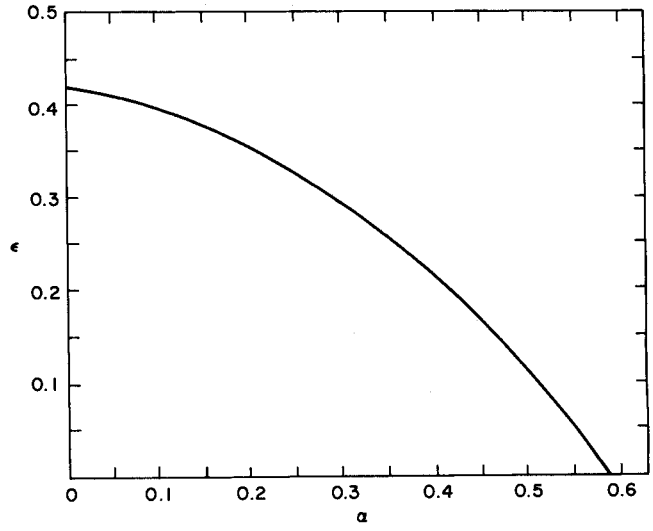

Fig. 3. ϵ vs. α, $n = 1/3$, $D_R = 5.85$, $T_0 = 1$.

as 55 to 83% of the value measured in shear in order to obtain agreement with the experimental data. These results are unchanged by the present analysis, for despite the fact that polyethylene is shear thinning, the computation of the modulus is unchanged for $n \neq 1$ in the high stress (small ϵ/α) limit because the deformation rate is constant over the entire length of the filament.

Zeichner (1973) has reported data on the isothermal spinning of polystyrene which provide a test of the theory in the intermediate force region. The spinning conditions are given in Table 1. The variation in the stretch rate was from 0.03 to 0.18 s^{-1}, and in this range the shear viscosity data were fit with a power law index in the range $0.31 \leq n \leq 0.36$. The value of n for calculation was taken as $n = 1/3$. The shear modulus varied by about 50% over this range of stretch rate, with the

Table 1. Spinning Conditions for Polystyrene, Data of Zeichner (1973)

$K = 4.7 \times 10^4$ poise · $s^{-2/3}$ $\quad n = 1/3$
$G = 2.6 \times 10^4 - 3.8 \times 10^4$ dynes/cm²
$v_o = 0.29$ cm/s $\quad L = 20$ cm
Mass flow rate = 0.0328 g/s
$F = 11\,700$ dynes
Temperature = 170°C
$0.21 \leq \alpha \leq 0.31$ $\quad \epsilon = 0.08$

lower value corresponding to the higher stretch rate at the take-up. The shear modulus and viscosity used by Zeichner are somewhat different from those measured for the same material by Richardson et al. (1975), reflecting either differences in measuring technique or perhaps different thermal histories. The calculations reported here use the data in Zeichner's thesis. The value of α in this experiment is then approximately in the range $0.21 \leq \alpha \leq 0.31$, and ϵ is approximately 0.08. The value of ϵ is probably uncertain to within about 20% even without considering possible inaccuracies in the viscosity measurement, since the force is read from a strip chart recorder with a condensed scale and is the least accurate of the reported numbers; some of Zeichner's strip chart data are shown in Fisher and Denn (1975).

The measured velocity profile and ϵ (or reciprocal force) represent independent experimental checks on the theory. It is evident from Figures 2 and 3 that neither the velocity nor the force data are adequately represented by the viscoelastic equations for α in the range 0.21 to 0.31, though the result is somewhat better than the inelastic power law solution, $\alpha = 0$. On the other hand, a value of α somewhere between 0.4 and 0.5 gives a reasonable representation of the velocity profile, while the measured force ($\epsilon \approx 0.08$) requires a value of α of slightly greater than 0.5. Better agreement with both measurements can be obtained for α of order 0.5 by taking T_0 to be greater than unity, but it is not apparent that such precision is warranted by either the data or the theory. In general, we can conclude that the theory is in agreement with experiment provided that the shear modulus in the extensional flow is from 40 to 80% of the value in shear flow with the viscosity unchanged, or if the viscosity is increased and the modulus decreased by lesser amounts. This conclusion is in striking agreement with the conclusion of Denn et al. (1975) regarding polyethylene. Furthermore, it agrees with results of Richardson et al. (1975) on uniform transient extension of the same polystyrene. Richardson et al. obtain an opposite result for polyethylene, however, though their conclusions are for smaller strains and may not be comparable.

The fact that the measured shear modulus is different in shear and extensional flows should be expected. A single relaxation time model involves an averaging over a relaxation spectrum, and the average is of necessity a different one in different flow fields. An extensional flow is transient in a Lagrangian sense, and the faster relaxation modes should be more easily excited, leading to an apparent modulus which is larger than the shear averaged value. The relevant portion of the spectrum will shift with time, leading to some dependence of apparent modulus on residence time. This effect is discussed in the context of a transient creep experiment by Acierno et al. (1976).

STABILITY TO INFINITESIMAL DISTURBANCES

The stability of a steady state to infinitesimal disturbances is determined by linearizing the system partial differential equations about the steady state. The procedure is a standard one and is discussed in texts such as Denn (1975). We seek separation-of-variables solutions to the linear partial differential equations in the form

$$a(\xi, \theta) - a_s(\xi) = \sum_{m=1}^{\infty} e^{\lambda_m \theta} \phi_m(\xi) + e^{\lambda_m^* \theta} \phi_m^*(\xi) \tag{23a}$$

$$u(\xi, \theta) - u_s(\xi) = \sum_{m=1}^{\infty} e^{\lambda_m \theta} \psi_m(\xi) + e^{\lambda_m^* \theta} \psi_m^*(\xi) \tag{23b}$$

$$T(\xi, \theta) - T_s(\xi) = \sum_{m=1}^{\infty} e^{\lambda_m \theta} \Gamma_m(\xi) + e^{\lambda_m^* \theta} \Gamma_m^*(\xi) \tag{23c}$$

$$[T(\xi, \theta) - P(\xi, \theta)] - [T_s(\xi) - P_s(\xi)] = \sum_{m=1}^{\infty} e^{\lambda_m \theta} \Pi_m(\xi) + e^{\lambda_m^* \theta} \Pi_m^*(\xi) \tag{23d}$$

* denotes the complex conjugate. It is convenient here to work with the difference $T - P$ rather than with P. The separation of variables solution leads to a set of linear ordinary differential equations:

$$\frac{d\phi_m}{d\xi} = \frac{1}{u_s}\left[\frac{\psi_m}{u_s^2}\frac{du_s}{d\xi} - \frac{1}{u_s}\frac{d\psi_m}{d\xi} - \left(\lambda_m + \frac{du_s}{d\xi}\right)\phi_m\right] \tag{24a}$$

$$\frac{d\psi_m}{d\xi} = \frac{\lambda_m(\Pi_m + u_s^2\phi_m) + \alpha\left(\frac{du_s}{d\xi}\right)^{n-1} + (2\Pi_m - 3\Gamma_m)\frac{du_s}{d\xi}}{n\left[3\frac{\epsilon}{\alpha} + 3T_s - 2u_s\right]} \tag{24b}$$

$$\frac{d\Gamma_m}{d\xi} = \frac{1}{u_s}\left\{\frac{1}{\alpha\left(\frac{du_s}{d\xi}\right)^{n-1}}\left[\frac{d\psi_m}{d\xi}\left(\frac{du_s}{d\xi}\right)^{n-2}\left(2\epsilon n\frac{du_s}{d\xi} - (n-1)\alpha u_s\frac{dT_s}{d\xi}\right) - \Gamma_m\right] + 2\Gamma_m\frac{du_s}{d\xi} + 2nT_s\frac{d\psi_m}{d\xi} - \psi_m\frac{dT_s}{d\xi} - \lambda_m\Gamma_m\right\} \tag{24c}$$

$$\frac{d\Pi_m}{d\xi} = -\frac{1}{u_s}\left\{\frac{1}{\alpha\left(\frac{du_s}{d\xi}\right)^{n-1}}\left[\Pi_m - 3\epsilon n\left(\frac{du_s}{d\xi}\right)^{n-1}\frac{d\psi_m}{d\xi}\right] + \left(\lambda_m + \frac{du_s}{d\xi}\right)\Pi_m + (\psi_m - 3\Gamma_m)\frac{du_s}{d\xi} + [n(2u_s - 3T_s) - u_s]\frac{d\psi_m}{d\xi}\right\} \tag{24d}$$

The boundary conditions follow from Equation (14):

$$\phi_m(0) = \psi_m(0) = \Gamma_m(0) = 0 \qquad \psi_m(1) = 0 \tag{25}$$

Equations (24) and (25) form a set of linear homogeneous equations with homogeneous boundary conditions. Nontrivial solutions will exist only for certain eigenvalues, λ_m. The structure of the solution in Equation (23) is such that infinitesimal disturbances will decay and the system will return to the steady state if

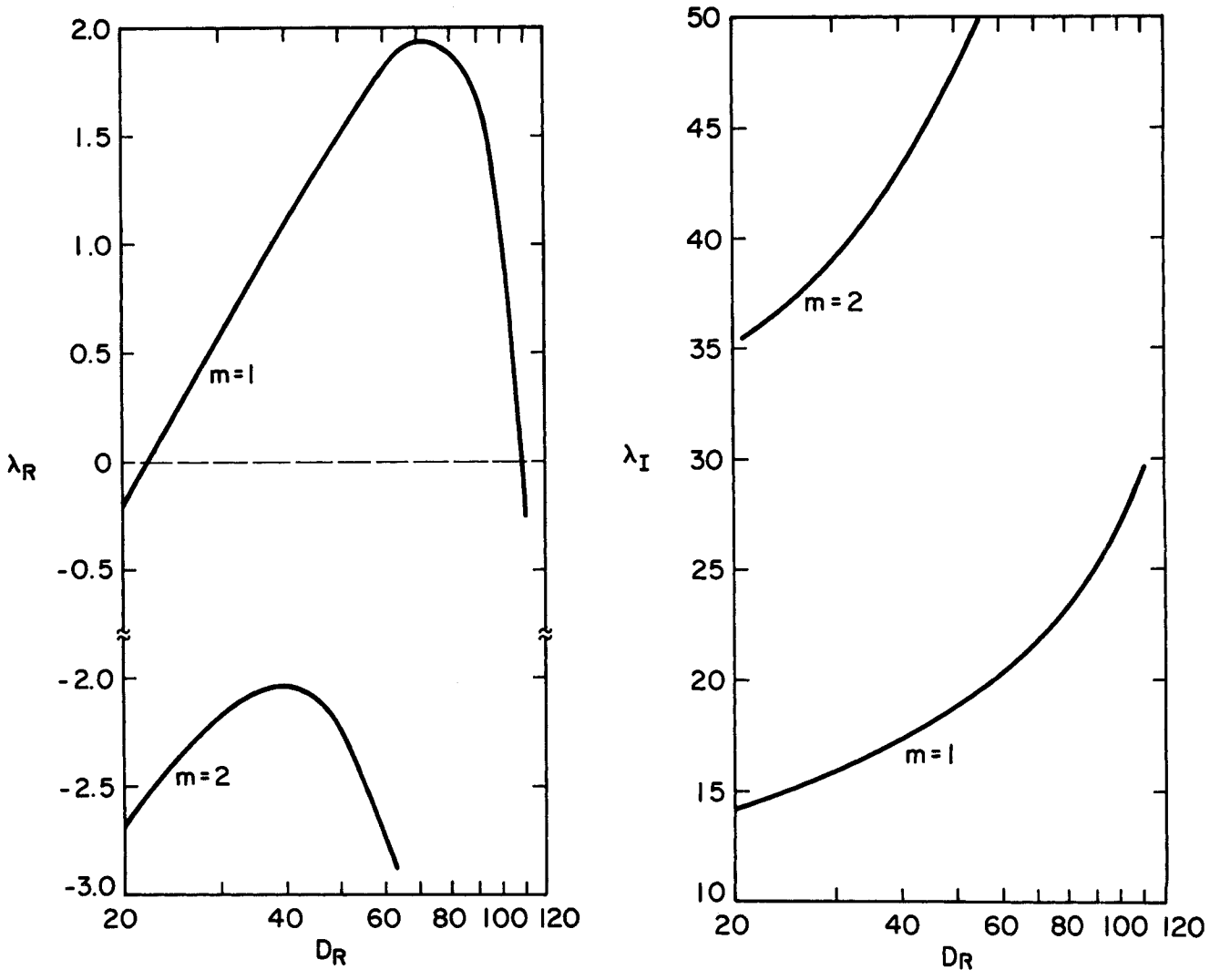

Fig. 4. Variation of the (a) real and (b) imaginary parts of the first two eigenvalues with draw ratio, $n = 1$, $\alpha = 0.006$, $T_0 = 1$.

the real parts of all λ_m are negative. If the real part of any λ_m is positive, then the disturbance will grow, and the steady state is absolutely unstable and cannot be maintained.

The solution to the eigenvalue problem was obtained by direct Kutta-Merson numerical integration of Equations (24), normalizing $\Pi_m(0)$ to unity and varying λ_m until the complex boundary condition $\psi_m(1) = 0$ was satisfied. The dependence of the first two eigenvalues on D_R is shown in Figure 4 for $n = 1$, $\alpha = 0.006$. This general behavior is typical of all n and all α. The first two eigenvalues are widely spaced, and the stability is determined by the eigenvalue of smallest absolute value. There is a critical draw ratio at which the real part of the first eigenvalue becomes positive, signaling a process instability. At a sufficiently high draw ratio, however, the real part of the first eigenvalue becomes negative again, indicating stability to infinitesimal disturbances at high draw ratio.

The results of the calculations are shown in Figure 5. There is a stability envelope for each value of n; the flow is stable to infinitesimal disturbances outside the envelope and unstable within. Numerical experimentation for $n = 1$ indicates that the envelope is relatively insensitive to T_o over the range $2/3 \leq T_o \leq 1\,000$. The envelope is also relatively insensitive to the presence of a secondary normal stress difference obtained by including the parameter ν of Denn et al. (1975); details of this latter calculation for a ratio of secondary to primary normal stress difference of -0.1 are given in Fisher (1975).

The unattainable region shown in Figure 5 corresponds to that region for which inequality (22) is violated, and steady state solutions to the spinning equations do not exist. By using $\alpha^{1/n}$ as the parameter, this region is the same for all n. For $\alpha \rightarrow 0$ the results of Pearson and Shah (1974) for an inelastic liquid are recovered. The critical draw ratio is relatively insensitive to α until $\alpha^{1/n} \sim 0.12$ $D_R{}^{-1}$, where D_R is the critical value for an inelastic liquid. Here, the spinning conditions are approaching the region of high tensile stress, the critical draw ratio begins to increase rapidly, and the envelope ultimately doubles back, all envelopes approaching a common high stress asymptote. Thus, there is a region of stability to infinitesimal disturbances at high draw ratio, and there is a value of α beyond which the fluid elasticity stabilizes all attainable flows. A result of this type for $n = 1$ was obtained by Zeichner (1973; Denn, 1975), but there are approximations in that analysis which are not valid over the entire parameter range, and the correct stability envelope differs somewhat from that calculation. A comparison is given by Fisher (1975).

The first onset of unstable flow is governed over a wide range of parameter values by the power law index of the shear viscosity, and the purely viscous response provides a lower estimate of the onset of instability in all cases. Agreement with experiment here is good. For

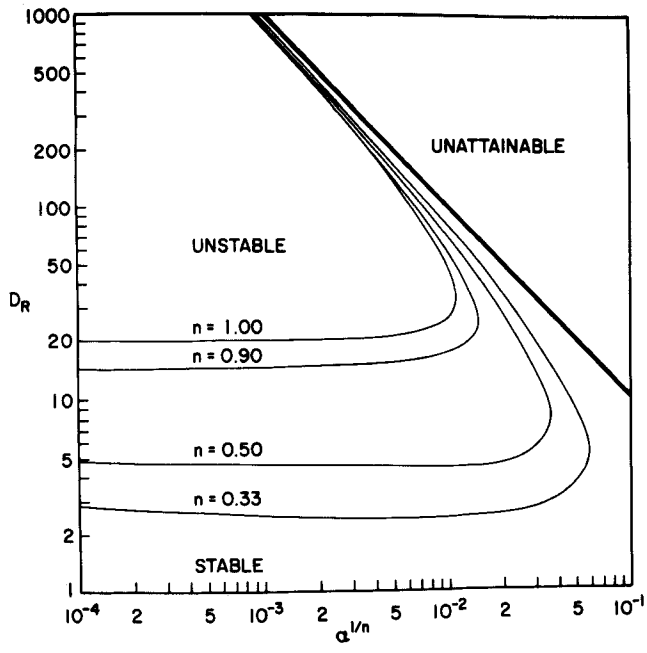

Fig. 5. Region of stability to infinitesimal disturbances.

the polystyrene studied by Zeichner with spinning conditions given in Table 1, with $n = 1/3$, the value of $\alpha^{1/n}$ is in the range 0.01 to 0.03 if the shear value of G is used or 0.06 to 0.13 if the value of G needed to match the steady state profile is used. The former places his experiments near the nose of the $n = 0.33$ curve, with a critical draw ratio in the range 3 to 5. The latter extends from the nose of the curve, with a critical draw ratio of about 6, into the stable region. The experimental critical draw ratio was 5.8. Data of Ishihara and Kase (1975*b*) on unstable spinning of PET at 300°C extrapolate to stabilization at a draw ratio of just above 20. The power law exponent of PET is close to unity, and the value of α in this experiment can be estimated at less than 5×10^{-3}, where the theoretical transition also occurs at a draw ratio of about 20. Donnelly and Weinberger (1975) have measured the onset of draw resonance in a polysiloxane of constant shear viscosity at a critical value D_R of slightly over 20. The relaxation time of this polymer is known to be small, and the small α result would be expected to apply.

Stability at large α is consistent with the known result that spinning can be stabilized by shortening the spin line length. (It should be borne in mind, however, that the assumption that the die swell is not affected by disturbances may be a particularly serious defect in application of the analysis to very short spin lines.) The existence of a stable region at very high draw ratio is of clear processing significance. An upper stable region has been observed by Lamb (1967), but his experiments are probably influenced by cooling, and the phenomenon may be different from the one described here. Other data by Ishihara and Kase provide a more direct verification of the prediction of the upper stable region, but it is more convenient to discuss these data following an analysis of the nonlinear dynamics of the spinning process.

Some brief comments are appropriate here regarding the applicability of an inelastic power law analysis of stability to infinitesimal disturbances proposed by Pearson and Shah (1974). To use their results, it is necessary that the apparent extensional viscosity in the steady state be fit by a power law, and that this power law index be used to predict stability. Zeichner's spinning data shown in Figure 2 can be fit by a power law index of 0.82, which is considerably larger than the proper value for the shear viscosity. An inelastic liquid with power law index 0.82 is predicted to become unstable at a draw ratio of 12, which is not in agreement with the data. Cruz-Saenz et al. (1975) have also found that the power law approach fails for a number of polymers, both extension thinning and extension thickening. On the other hand, the good agreement noted here is for a power law exponent obtained from shear measurements.

NONLINEAR DYNAMICS

Calculations of stability to infinitesimal disturbances can provide only a partial picture of the dynamical behavior of the process. Infinitesimal theory cannot determine the response of the system to finite disturbances when it is stable to infinitesimal disturbances, nor can it determine what the dynamical response will be when an unstable infinitesimal disturbance has grown to finite size. Without further analysis, therefore, it cannot be established that the stability envelope in Figure 5 defines the complete region of unstable behavior, nor can it be established that the instability represented there is truly the draw resonance instability which is observed experimentally.

The usual approach to the study of nonlinear dynamics is to expand the solution in the eigenfunctions of the linear system (Denn, 1975). The basic assumption is that the set of eigenfunctions is complete. Thus, we would seek solutions to Equations (10) through (14) in the form

$$a(\xi, \theta) = a_s(\xi) + \sum_{m=1}^{\infty} A_m(\theta)\phi_m(\xi) + A_m^*(\theta)\phi_m^*(\xi) \tag{26a}$$

$$u(\xi, \theta) = u_s(\xi) + \sum_{m=1}^{\infty} B_m(\theta)\psi_m(\xi) + B_m^*(\theta)\psi_m^*(\xi) \tag{26b}$$

$$T(\xi, \theta) = T_s(\xi) + \sum_{m=1}^{\infty} C_m(\theta)\Gamma_m(\xi) + C_m^*(\theta)\Gamma_m^*(\xi) \tag{26c}$$

$$[T(\xi, \theta) - P(\xi, \theta)] = [T_s(\xi) - P_s(\xi)] + \sum_{m=1}^{\infty} D_m(\theta)\Pi_m(\xi) + D_m^*(\theta)\Pi_m^*(\xi) \tag{26d}$$

ϕ_m, ψ_m, Γ_m, and Π_m are solutions to Equations (24). In the limit of small disturbances, the complex functions $A_m(\theta)$, $B_m(\theta)$, $C_m(\theta)$, $D_m(\theta)$ must reduce within a normalizing constant to $\exp(\lambda_m\theta)$, where λ_m is the corresponding eigenvalue of the linear system.

We now proceed in analogy to our earlier solution of the nonlinear dynamics of Newtonian spinning (Fisher and Denn, 1975), and we set $n = 1$ for simplicity. First, because the eigenvalues given in Figure 4 are widely spaced, we may assume that the first mode dominates the transient and neglect all $m > 1$ in the summations in Equations (26). Thus, we take

$$a(\xi, \theta) \approx a_s(\xi) + [A_R(\theta)\phi_R(\xi) - A_I(\theta)\phi_I(\xi)] \tag{27}$$

and similarly for the other variables, where the subscripts R and I refer to the real and imaginary parts of the complex functions, respectively, and the subscript 1 has been dropped for convenience. Next, the approximate solutions are substituted into the nonlinear partial differential equations. Equation (10), for example, becomes

$$\phi_R(\xi)\frac{dA_R}{d\theta} - \phi_I(\xi)\frac{dA_I}{d\theta} + A_R(\theta)\frac{du_s\phi_R}{d\xi} - A_I(\theta)\frac{du_s\phi_I}{d\xi} + B_R(\theta)\frac{da_s\psi_R}{d\xi} - B_I(\theta)\frac{da_s\psi_I}{d\xi} = \mathcal{R}_a \neq 0 \quad (28a)$$

or, symbolically

$$\phi_R(\xi)\frac{dA_R}{d\theta} - \phi_I(\xi)\frac{dA_I}{d\theta} + F_a(A, B, C, D, \phi, \psi, \Gamma, \Pi) = \mathcal{R}_a(\theta, \xi) \quad (28b)$$

$\mathcal{R}_a(\theta, \xi)$ is the residual, which reflects the fact that the approximate solution fails to satisfy the differential equation for each position and time. There are similar equations for the other three variables.

We obtain a set of ordinary differential equations for the $A_R(\theta)$, A_I, (θ), etc., by using Galerkin's method (Finlayson, 1972; Denn, 1975) in which the residual of each equation is made orthogonal to each approximating function for the variable defined by that equation. The continuity equation defines the area, and the momentum equation defines the velocity in this sense. Thus, setting $\mathcal{R}_a$ orthogonal to both $\phi_R(\xi)$ and $\phi_I(\xi)$ leads to

$$\left[\int_0^1 \phi_R^2(\xi)d\xi\right]\frac{dA_R}{d\theta} - \left[\int_0^1 \phi_R(\xi)\phi_I(\xi)d\xi\right]\frac{dA_I}{d\theta} + \int_0^1 F_a\phi_R(\xi)d\xi = 0 \quad (29a)$$

$$\left[\int_0^1 \phi_R(\xi)\phi_I(\xi)d\xi\right]\frac{dA_R}{d\theta} - \left[\int_0^1 \phi_I^2(\xi)d\xi\right]\frac{dA_I}{d\theta} + \int_0^1 F_a\phi_I(\xi)d\xi = 0 \quad (29b)$$

with similar equations for the other time variables. The complete set of eight equations is given by Fisher (1975).

The behavior of the viscoelastic system is much like that for the Newtonian fluid (Fisher and Denn, 1975). Below the critical draw ratio defined by infinitesimal theory, finite disturbances decay in time as long as they are not so large as to make the filament area go to zero in the first cycle. Above the critical value the sustained finite oscillations of draw resonance are obtained. For the viscoelastic fluid, the system again becomes stable to finite disturbances beyond the upper branch of the stability envelope, indicating that the infinitesimal theory does define the complete stability boundary.

The oscillations are most conveniently represented in terms of the ratio of maximum-to-minimum diameter of the drawn filament. Figure 6 shows such calculations, including points in both the lower and upper stable region, for $n = 1$, $\alpha = 0.006$. The two portions of the curve cannot be connected because the approximate theory breaks down at large amplitudes.

Ishihara and Kase (1975*b*) have reported data on draw resonance for two PET samples at 280°C for various spinning lengths. PET has a nearly shear independent viscosity, so the calculations for $n = 1$ should apply. PET-A, spun at $D_R = 48.6$, has an intrinsic viscosity of 0.563 and thus has a relaxation time (μ/G) of approximately 10^{-3} s (Gregory, 1973). PET-B, spun at $D_R = 50$, is not characterized in the paper, but it may be assumed to have similar properties. The data are shown in Figure 7, together with a computed curve for $\mu/G = 10^{-3}$ s, $D_R = 48$. The agreement at small spinning lengths

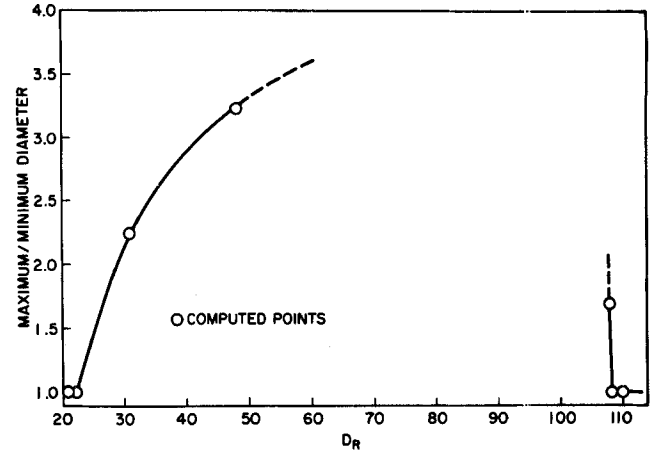

Fig. 6. Ratio of maximum to minimum diameter of the drawn filament as a function of draw ratio, $n = 1$, $\alpha = 0.006$, $\Gamma_0 = 1$.

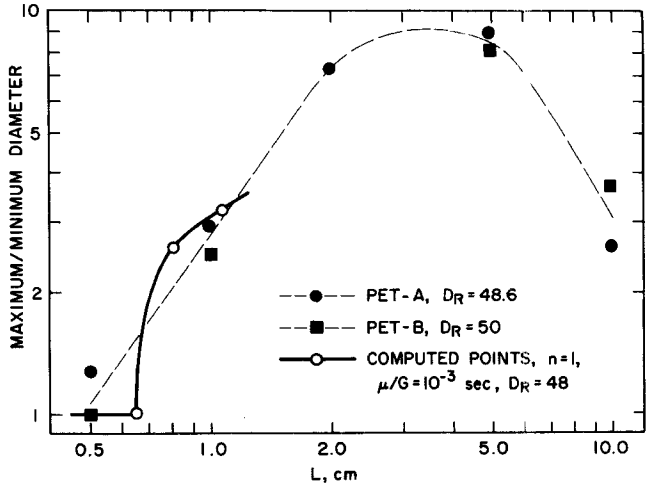

Fig. 7. Ratio of maximum to minimum diameter of the drawn filament as a function of length. Data of Ishihara and Kase (1975*b*) on two PET melts at draw ratios of 48.6 and 50. Computed curve for $n = 1$, $D_R = 48$, $\mu/G = 10^{-3}$ s.

between theory and this a priori calculation is excellent even without accounting for approximations in the theory and the difficulty of taking accurate data over such short spinning distances. The stabilization by decreasing L corresponds to approaching the neutral stability curve at fixed D_R by increasing α. The value $D_R = 48$ puts this meeting point on the upper branch of the stability envelope, providing strong experimental support of the predictive power of the theory. The apparent stabilization at longer lengths is probably a consequence of cooling and may correspond to the upper stable region reported by Lamb (1967).

CONCLUSION

The mechanics of isothermal melt spinning are accurately represented by solution of the continuity and momentum equations together with a Maxwell type of rheological model with deformation rate-dependent viscosity but constant modulus. The modulus used in this model must be somewhat different for polystyrene and polyethylene in an extensional flow than in a shear flow. Thus it appears that a widely useful viscoelastic model must include this dependence of the modulus on the degree of extension of the flow field. Some suggestions in this regard have been made (Kanel, 1972; Huilgol, 1975), but a definitive formulation does not exist.

The model equations predict the stability and transient response of the isothermal spinning process extremely

well and can be used for predictive purposes within the accuracy of the estimate of the modulus. The first onset of draw resonance is insensitive to the value of the modulus over a wide parameter range, so the dependence of the modulus on the type of flow field is not very important, and normal stress or dynamic measurements of G should suffice. The location of the nose of the stability envelope and the upper stable region depend on $G^{1/n}$. For a constant viscosity liquid with $n = 1$, these points will not vary considerably with changes of the order of 20 to 50% in G; this may be the reason for the good agreement with PET data in Figure 7 with a shear measurement of μ/G. For a highly shear thinning liquid, the upper stable region is quite sensitive to G. For polystyrene with $n = 1/3$, for example, the critical draw ratio or spinning length for stabilization varies with G^3, and uncertainties in the modulus can have a marked effect on the results.

ACKNOWLEDGMENT

This work derives from a preliminary study in the thesis of G. R. Zeichner (1973), which was advised jointly by B. E. Anshus, M. M. Denn, and A. B. Metzner, and we are grateful to these colleagues for their help both prior to and during the present study, as well as to K. Wissbrun and J. C. Miller for first introducing our group to the problem of draw resonance. C. J. S. Petrie has been a valuable colleague in our work in spinning, and he has been most helpful to us in this study. J. A. Spearot and G. Marrucci both suggested the probable effect on the modulus of a distribution of relaxation times. We are also grateful to C. B. Weinberger and S. Kase for useful discussion and for providing us with their experimental data prior to publication. This work was supported by the National Science Foundation under Grant GK-43303.

NOTATION

a = dimensionless area
A = area
A_m = function of time in area expansion
B_m = function of time in velocity expansion
C_m = function of time in T expansion
D_m = function of time in $T - P$ expansion
D_R = draw ratio
F = force
G = shear modulus
K = coefficient in power law viscosity
L = filament length
n = power law index
P = dimensionless extra stress in x direction
Q = volumetric flow rate
$\mathcal{R}$ = residual
t = time
T = dimensionless extra-stress in z-direction
u = dimensionless axial velocity
$\mathbf{v}, v_i$ = velocity vector
x = direction normal to extension
z = axial direction
$()^T$ = transpose
* = complex conjugate
∇ = gradient operator
II_d = second invariant of $\nabla\mathbf{v} + (\nabla\mathbf{v})^T$

Greek Letters

α = dimensionless rheological parameter
$\dot{\gamma}_s$ = shear rate
Γ_m = eigenfunction for T
ϵ = dimensionless reciprocal force
θ = dimensionless time
λ_m = eigenvalue
μ = viscosity
ξ = dimensionless axial position
Π_m = eigenfunction for $T - P$
τ, τ_{ij} = extra stress tensor
τ_o = initial value of τ_{zz}
ϕ_m = eigenfunction for a
ψ_m = eigenfunction for u

Subscripts

I = imaginary part of complex number
o = initial value
R = real part of complex number
s = steady state

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Manuscript received August 22, 1975; revision received November 25 and accepted November 28, 1975.

A Mechanism Supported by Extensive Experimental Evidence to Explain High Heat Fluxes Observed During Nucleate Boiling

This paper attempts to elucidate the mechanism responsible for high heat transfer rates occurring in nucleate boiling when liquid films exist on the heating surface. High-speed cinematography and simultaneous transient surface temperature measurements provide a basis for describing the mechanism. In a liquid film, bubbles grow and detach rapidly. The film is quickly renewed. A liquid microlayer exists beneath a bubble its entire life. Conditions are very favorable for rapid evaporation from the microlayer so that heat transfer is rapid. A number of independent observations by other investigators working on diverse problems are cited in support of the mechanism.

RUSSELL MESLER

Department of Chemical and Petroleum Engineering
University of Kansas
Lawrence, Kansas 66045

SCOPE

Ebullition sometimes occurs in a thin liquid film on a surface during nucleate boiling. Kusada and Nishikawa (1967) studied nucleate boiling occurring in thin liquid films. They observed that dry out often occurred with water films thinner than 1 mm, and they also generally observed a maximum heat transfer coefficient in this range of thicknesses. Toda and Uchida (1973) have reported achieving exceptionally high heat transfer rates to films of water 0.2 to 0.7 mm thick flowing over a surface at 2 to 10 m/s. Kirby and Westwater (1965) report that such ebullition occurred at high heat fluxes beneath a frothy mixture in their pool boiling studies of methanol and carbon tetrachloride. Katto et al. (1970) report similar observations for water. Numerous studies of boiling in falling liquid films have shown that much higher heat transfer coefficients occur than with pool boiling.

Bubbles growing in a thin liquid film have received little previous study, yet they possibly account for the exceptionally good heat transfer. These bubbles differ markedly from those growing in normal pool boiling in their shorter life and rapid detachment. The significance of these differences has received little consideration.

Ebullition during nucleate boiling generally cools the surface upon which the bubble grows by the evaporation of a thin liquid layer beneath the bubble. This has been called microlayer evaporation. There has as yet been no report on whether microlayer evaporation occurs during ebullition in thin liquid films.

Bubbles rising through liquid to break at a free surface have been studied by several investigators because they entrain liquid droplets on bursting. Such studies provide a clue to understanding bubbles growing in thin liquid films and how they achieve such rapid departure.

High-speed cinephotography has proven to be a most informative technique for studying bubbles during nucleate boiling. In its use, an unobstructed profile view of the bubble is very helpful, but obtaining such a view is a formidable problem. A serendipitous experiment reported by Williams and Mesler (1967) offered a unique solution. Boiling from an artificial nucleation site produces bubbles so readily from the site that they tend to grow in the thin liquid layer left by a departing bubble.

A useful method of studying microlayer evaporation is to measure the transient surface temperature beneath a growing bubble. A special tiny fast response surface thermocouple has proven useful for this purpose. It must be small enough so that a bubble covers it as the bubble grows.

The relation of transient surface temperature to bubble growth is of vital importance in the investigation. Such in-